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CHAPTER 6

Describing Polygons with Systems of Inequalities

With two linear inequalities, we can define a wedge-shaped region in the coordinate plane. With three or more linear inequalities, we can define a finite, polygon-shaped region—three inequalities for a triangle, four for a quadrilateral, and so on. Since in most applications we will want to include the border of the region, the inequalities will normally be non-strict. That is, we will usually (but not always) use ≤ and ≥ rather than < and >.

EXAMPLE 1 Graph systems of inequalities

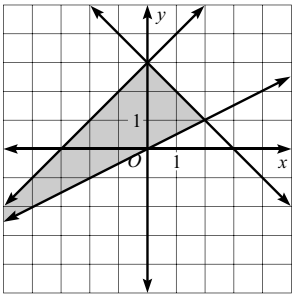
Graph each system of inequalities.

a. $x + y \le 3$
$y \le x + 3$
$y \ge \frac{1}{2}x$

b. $x \ge -2$
$x \le 3$
$y \le 4$
$y \ge \frac{2}{3}x - 2$
$y \ge -2x - 4$

Solution:

a.



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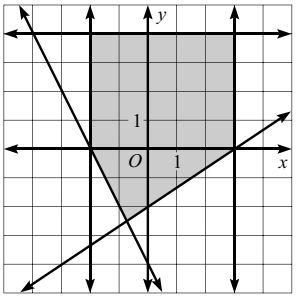

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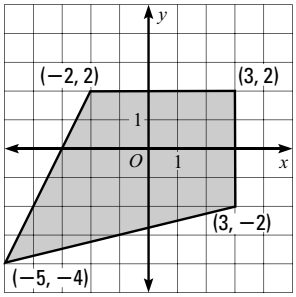
Sometimes, we will need to write the system of inequalities using other information, such as the vertices of the polygon. In that case, we take two vertices at a time and use the procedure for finding the equation of a line through two points. We then convert each equation into the appropriate inequality.

EXAMPLE 2 Describe a region with a system of inequalities

Write a system of inequalities to describe the quadrilateral region with corners at (−5, −4), (−2, 2), (3, 2), and (3, −2).

Solution:

We begin by graphing the region.



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Taking two adjacent vertices at a time, we find the equations for the lines through them. The slope of the line through $(-5, -4)$ and $(-2, 2)$ is $\frac{2-(-4)}{-2-(-5)} = \frac{6}{3} = 2$, and the line through the two points is given in point-slope form as $y - 2 = 2(x + 2)$ or in slope-intercept form as $y = 2x + 6$. The slope of the line through $(-2, 2)$ and $(3, 2)$ is $\frac{2-2}{3-(-2)} = \frac{0}{5} = 0$, and the line through the two points is $y = 2$.

Using the pairs of points $(3, 2)$, $(3, -2)$ and $(-5, -4)$, $(3, -2)$, we find the two remaining equations: $x = 3$ and $y = \frac{1}{4}x - \frac{11}{4}$.

Now we write the corresponding inequalities by substituting $\geq$ or $\leq$ for $=$ in each equation, depending on which side of the line represents the interior of the polygon.

Since the quadrilateral lies *below* the line $y = 2x + 6$, we write $y \leq 2x + 6$.

Since the quadrilateral lies *below* the line $y = 2$, we write $y \leq 2$.

Since the quadrilateral lies *to the left of* the line $x = 3$, we write $x \leq 3$.

Since the quadrilateral lies *above* the line $y = \frac{1}{4}x - \frac{11}{4}$, we write $y \geq \frac{1}{4}x - \frac{11}{4}$.

The points that satisfy all four inequalities make up the quadrilateral region. In other words, the quadrilateral region with vertices at $(-5, -4)$, $(-2, 2)$, $(3, 2)$, and $(3, -2)$, including the boundaries, is the graph of the system of inequalities. ■

Graphs of polygonal regions, defined by inequalities, are used in a type of application problem called **linear programming**. The variables x and y typically represent different resources in some process, and the inequalities represent limits on how much of each resource can be used.

EXAMPLE 3 Graph a system of inequalities

A specialty store plans to sell large bags of nuts containing some mix of peanuts and cashews. The store buys its nuts wholesale at \$0.50 per pound for peanuts and \$2 per pound for cashews. Each bag should contain at least 0.5 pound of each type of nut, but one bag's worth of nuts should not cost the store more than \$4. Using x for pounds of peanuts and y for pounds of cashews, write a system of inequalities that defines the possible amounts of peanuts and cashews that go into each bag of mixed nuts. Graph the system.

Solution:

Since there must be at least 0.5 pound of each type of nut, two of the inequalities are $x \geq 0.5$ and $y \geq 0.5$.

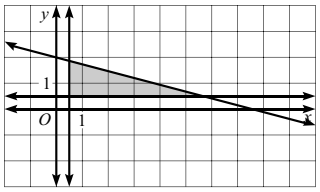
The third inequality comes from the limit on the store's total cost per bag: $0.5x + 2y \leq 4$, or in slope-intercept form, $y \leq -0.25x + 2$.

We graph the system of three inequalities.

$x \geq 0.5$

$y \geq 0.5$

$y \leq -0.25x + 2$ ■



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If Example 3 were a typical linear programming problem, a possible question might be "Which mix of nuts will produce the maximum profit per bag?" The answer would depend on how much customers were willing to pay for a given mix of nuts.

Suppose that customers only cared about the amount of nuts, and were willing to pay the same per pound regardless of the cashews-to-peanuts ratio. Profit would be maximized by putting in the minimum amount of cashews and then filling the bag up to the limit with peanuts. The lower right corner of the solution graph shows that a bag would contain 0.5 pound of cashews and 6 pounds of peanuts.

Practice

Graph each system of inequalities.

1. $x \le 0$
$y \ge -3$
$2x - y \ge -3$

2. $x + y \le 0$
$y \ge -2$
$4x - y \ge -10$

3. $x + 2y < 2$
$3x - 2y \ge 6$
$5x + 2y \ge -6$

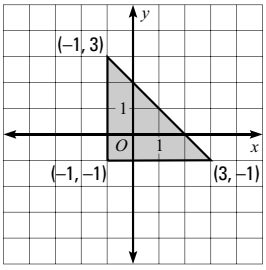
4. $x \ge -2$
$y \ge -3$
$y \le 1$
$3x + y \le 6$

5. $x \le 3$
$y \le 4x + 2$
$2x - 3y \ge -3$
$x + 3y \ge -6$

6. $y \le -3x + 8$
$4y + x \le 10$
$y \ge -5x - 7$
$4y - x > -7$

Use inequalities to describe each graph. Write your inequalities in slope-intercept form.

7.



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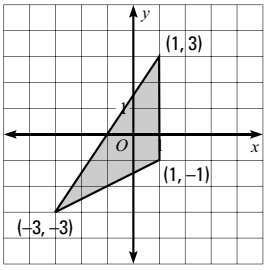

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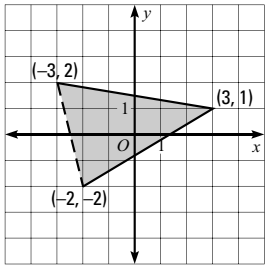

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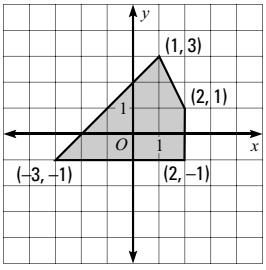

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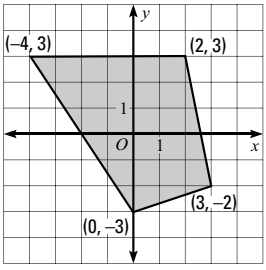

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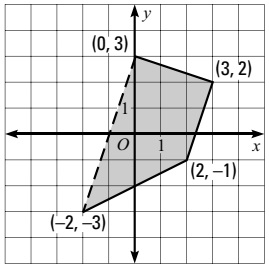

13. In Example 3, suppose that all customers really care about (and are willing to pay for) is cashews. Which mix of nuts would maximize the profit per bag?

14. **Writing** When a solid boundary meets a dashed boundary, is the corner point part of the solution of the system? Why or why not?